[This question paper contains 4 printed pages.]

**Your Roll No..............**

**Sr. No. of Question Paper : 5806** **H**

Unique Paper Code : 237501

Name of the Paper : Paper STH-501 / Statistical Inference-I

Name of the Course : **B.Sc. (Honours) Statistics**

Semester : V

Duration : 3 Hours Maximum Marks : 75

**Instructions for Candidates**

1. Write your Roll No. on the top immediately on receipt of this question paper.

2. Attempt **SIX** questions in all, selecting **THREE** questions from each Section.

## Section – I

1. State and prove Cramer-Rao inequality. (i) By making a suitable assumption, obtain the alternative form of the inequality, (ii) Also obtain the form of Cramer-Rao inequality in case $X_1, X_2, \ldots, X_n$ are i.i.d. random variables with common p.d.f. $f_\theta(x)$. (12½)

2. (a) If $T_1$ and $T_2$ are two unbiased estimators of $\gamma(\theta)$ with variances $\sigma_1^2$, $\sigma_2^2$ respectively and correlation coefficient $\rho$, then what is the best unbiased linear combination of $T_1$ and $T_2$ and what is the variance of such a compound. If $T_1$ is a minimum variance unbiased estimator and $T_2$ is any other unbiased estimator for $\gamma(\theta)$ with efficiency $e_\theta$, then show that $\rho = \sqrt{e_\theta}$.

(b) In random sampling from $N(\mu, \sigma^2)$ population, where both $\mu$ and $\sigma^2$ are unknown, obtain the estimator of the form $aS^2$ for $\sigma^2$, which has the smallest mean-square error. (9,3½)

3. Define a sufficient statistic. Let $X_1, X_2, \ldots, X_n$ be a random sample from $U(0,\theta)$ population. Obtain MVUE for the parameter $\theta$. Compute the reciprocal of $nE\left[\left\{\frac{\partial \log f(x,\theta)}{\partial \theta}\right\}^2\right]$ and compare this with the variance of MVUE. (12½)

4. Define an unbiased and a consistent estimator for parameter $\gamma(\theta)$. State and prove sufficient conditions for estimator T to be consistent for $\gamma(\theta)$. Show with the help of an example that an estimator may be (i) consistent but not unbiased (ii) unbiased but not consistent, in estimating the unknown parameter $\gamma(\theta)$. (12½)



**Section – II**

5. (a) Explain the principle of maximum likelihood estimation.

(b) Show with the help of an example that an ML estimator may not be unique.

(c) Show that the most general form of a continuous distribution for which the sample harmonic mean is the ML estimator of a parameter $\theta$ has p.d.f.

$$f(x,\theta) = \exp\left[\frac{1}{x}\{\theta A'(\theta) - A(\theta)\} - A'(\theta) + C(x)\right]$$

where $A(\theta)$ and $C(x)$ are arbitrary functions of $\theta$ and x respectively. (4,4,4½)

6. (a) Obtain estimates for the parameters $\theta_1$ and $\theta_2$ for the probability mass function:

$$p(x) = \frac{1}{2}\cdot\frac{e^{-\theta_1}\theta_1^x}{x!} + \frac{1}{2}\cdot\frac{e^{-\theta_2}\theta_2^x}{x!}, \quad x = 0, 1, 2, \ldots\ldots.$$

by the method of moments.

(b) Explain the method of minimum $\chi^2$ for estimating unknown parameters. Show that for large n, the minimum $\chi^2$ and ML methods of estimation give identical equations for determining the estimates. Explain modified minimum $\chi^2$ method. (5,7½)

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7. (a) Obtain $100(1-\alpha)\%$ confidence interval for the population correlation coefficient $\rho$ when a random sample of size n has been drawn from bivariate normal population.

(b) Develop a general method for constructing confidence intervals. Consider a random sample of size n from rectangular distribution with p.d.f.

$$f(x,\theta)=\frac{1}{\theta},\ 0\le x\le\theta.$$

Show that R and $R/\xi$ are the confidence limits for $\theta$ with confidence coefficient $(1-\alpha)$, where R is the sample range and $\xi$ satisfies the equation $\xi^{n-1}[n-(n-1)\xi]=\alpha$ (5,7½)

8. Write short notes on any **Two** of the following :

(i) General form of the distribution admitting sufficient statistic

(ii) Rao-Blackwell theorem

(iii) Properties of ML estimators (12½)